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Entangling two cavity modes via a two-photon process

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Abstract

We propose a scheme for entangling two field modes in two high-Q optical cavities. Using a virtual two-photon process, our scheme provides us a new kind of nonlinear interaction among the cavity modes *that is very different from the Kerr interaction*. Through this interaction maximally entangled states for the two modes may be achieved. The *ideal implementation* of our proposal requires no real population transitions of atomic internal states, hence it is immune to atomic decay.

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(Some figures in this article are in colour only in the electronic version)

Entanglement is one of the most characteristic features of quantum systems and lies at the heart of the difference between the quantum and classical multi-particle world. It is the phenomenon that enables quantum information processing and computing [1]. Beyond these and other related applications, complex entangled states, such as the GHZ triplets of particles [2], can be used for tests of quantum non-locality [3]. Moreover, the relaxation dynamics of larger entangled states sheds light on the decoherence process and on the quantum–classical boundary [4]. There are a lot of proposals devoted to the preparation of quantum entangled states [2, 3, 5–9], among them the ideas for photon down-conversion process [3], with trapped ions [6], for cavity quantum electrodynamics [7], with macroscopic objects [8] or for an optical fibre [9] have been realized experimentally.

In the latter case, the entanglement results from the nonlinear interaction between the two modes in an optical fibre. This is closely connected with the recent advance of enhancing nonlinear coupling via the electromagnetically induced transparency (EIT) mechanism [10]. The measured value of the $\chi^{(3)}$ parameter is up to six orders of magnitude larger than usual [11]. This has opened the door for the application of this kind of nonlinear process to quantum information processing even for the very low photon-number case [12]. In fact, there are

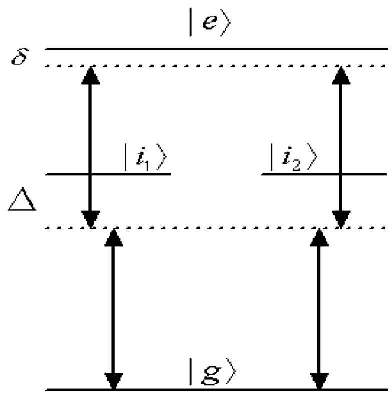

Figure 1. A four-level atom interacting with two cavity quantum fields. Both cavity fields are detuned from atomic resonance by $\Delta = \Omega - \omega_i$ and $\delta = 2\Omega - \omega_e$.

several proposals for exploiting huge Kerr nonlinearities to perform computation and quantum teleportation [13, 14] or for quantum non-demolition measurements [15]. Apart from the Kerr nonlinearity, the experimental achievement of atomic Bose–Einstein condensation (BEC) also provides us a chance to create many-particle entanglement with nonlinear interaction [16–23]. All these show that the nonlinear interaction between different quantum modes is a valuable resource for quantum information processing.

In this paper, we present a new theoretical scheme for entangling two quantum modes in two high-Q optical cavities. Through a virtual two-photon process, an effective nonlinear interaction between the two modes can be established. This interaction is quite different from that in the EIT mechanism. By using the virtual two-photon process, our new protocol significantly reduces the effect of atomic spontaneous emission during the entanglement preparation process.

Our system consists of two optical cavities as in [24] and an atom system surrounded by the two optical cavities. The axes of the two cavities are perpendicular to each other; the internal structure of the atom is depicted in figure 1.

The atom is assumed to make two-photon transitions of frequency ω_e between the nondegenerate state $|g\rangle$ with energy $\omega_g = 0$ and the excited state $|e\rangle$. The transitions are mediated by two intermediate degenerate levels $|i_1\rangle$ and $|i_2\rangle$ (with energy ω_i): the frequencies for transitions $|g\rangle \longleftrightarrow |i_1\rangle$ (or $|i_2\rangle$) and $|i_1\rangle$ (or $|i_2\rangle$) $\longleftrightarrow |e\rangle$ are $\Omega - \Delta$ and $\Omega + \Delta - \delta$, respectively. With this notation, the system can be described by

$$\begin{aligned} H = {}& \hbar\Omega_a a^\dagger a + \hbar\Omega_b b^\dagger b + \hbar g_c(|g\rangle\langle i_1|a^\dagger + |i_1\rangle\langle e|a^\dagger + \text{h.c.}) \\ & + \hbar g_c(|g\rangle\langle i_2|b^\dagger + |i_2\rangle\langle e|b^\dagger + \text{h.c.}) + \hbar\omega_i|i_1\rangle\langle i_1| + \hbar\omega_e|e\rangle\langle e| + \hbar\omega_i|i_2\rangle\langle i_2| \end{aligned} \quad (1)$$

where a (b) and $a^\dagger$ ($b^\dagger$) are the annihilation and creation operators for the cavity mode a (b) with frequency Ω_a (Ω_b), respectively, g_c is the coupling constant of the atom to the cavity mode a (b) driving the transition $|g\rangle \longleftrightarrow |i_1\rangle$ or $|i_1\rangle \longleftrightarrow |e\rangle$ ($|g\rangle \longleftrightarrow |i_2\rangle$ or $|i_2\rangle \longleftrightarrow |e\rangle$). We will not consider the position dependence of the cavity–atom coupling $g_c(\vec{r})$, a good approximation in the Lamb–Dicke limit. For simplicity, we assume $\Omega_a = \Omega_b = \Omega$ hereafter. Our scheme works in the following limit: (1) both the cavity modes a and b are strongly detuned, i.e., $\Delta = \Omega - \omega_i \gg g_c$, $\Delta \gg \delta$ and $\delta \gg |g_c|^2/\Delta$; and (2) the cavity decay rate $\kappa \ll |g_c|^2/\Delta$ as required for the high-Q optical cavity. Although the limit of $g_c^2 \gg \Gamma\kappa$ with Γ the atomic decay rate is a challenging pursuit [25], it could nevertheless be expected to be reachable with optical cavity QED based systems soon [26]. We will discuss this issue again at the end of this paper. Because these transitions $|g\rangle \longleftrightarrow |i_1\rangle$, $|i_1\rangle \longleftrightarrow |e\rangle$, $|g\rangle \longleftrightarrow |i_2\rangle$ and $|i_2\rangle \longleftrightarrow |e\rangle$ driven by the two cavity modes are far off-resonant, we may adiabatically eliminate the intermediate states $|i_1\rangle$ and $|i_2\rangle$ independently. The Hamiltonian (1) then takes the following form [27, 28]:

$$\mathcal{H} = \hbar\omega a^\dagger a + \hbar\omega b^\dagger b + \hbar\lambda(|g\rangle\langle e|a^{\dagger 2} + |e\rangle\langle g|a^2) + \frac{\omega_A}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + \hbar\lambda(|g\rangle\langle e|b^{\dagger 2} + |e\rangle\langle g|b^2) \quad (2)$$

with $\omega = \Omega + 2\frac{|g_c|^2}{\Delta}$, $\Delta = \Omega - \omega_i$, $\lambda = \frac{|g_c|^2}{\Delta}$, $\omega_A = \omega_e - \omega_g$. This is the Hamiltonian which is broadly used to describe the two-photon process, and has received extensive study during the last decades, for instance, the experimental realization of a two-photon cascade micromaser [29], the generation of squeezing amplification [30] and the creation of entangled states [31]. Our proposal works with a new mechanism different from that by using very high Kerr coupling. The coupling between the two cavity modes in our protocol, to be discussed below, results from virtual two-photon processes.

In the limit $\delta \gg \lambda$, i.e. $(2\omega - \omega_A) \gg |g_c|^2/\Delta$, the two-photon process is off-resonance; we may adiabatically eliminate the atom from the system. The Hamiltonian (2) then takes the following form in the interaction picture:

$$H_{\rm eff} = \hbar\frac{|\lambda|^2}{\delta}(a^{\dagger 2}a^2 + b^{\dagger 2}b^2 + a^{\dagger 2}b^2 + b^{\dagger 2}a^2). \quad (3)$$

In the derivation of the Hamiltonian (3), the atom is initially assumed to be in its ground state $|g\rangle$. The two-mode states will be defined in terms of the usual two-mode Fock states $|m, n\rangle = |m\rangle_a \otimes |n\rangle_b$ with m (n) photons in mode a (b). First we consider the simple case where there are only two photons in the mode a while the cavity mode b is initially in vacuum. The Hamiltonian (3) for this simple case is equivalent to

$$H_{\rm eff} = 2\hbar\frac{|\lambda|^2}{\delta}\left(|E\rangle_a\langle E| + |E\rangle_b\langle E| + \sigma_a^+\sigma_b^- + \sigma_a^-\sigma_b^+\right)$$

with definition $|E\rangle_x = |2\rangle_x$ $(x = a, b)$ and $\sigma_x^\pm$ is a linear combination of the Pauli operators for mode x, i.e. $\sigma_x^{+(-)} = \frac{1}{2}(\sigma_x \pm {\rm i}\sigma_y)$. It shows that after the interaction time $T_0 = \delta\pi/8|\lambda|^2$, the two cavity modes evolve to a maximal entangled state $\frac{1}{\sqrt{2}}(|0E\rangle + |E0\rangle)$ leaving the atom in its ground state $|g\rangle$. We would like to note that the nonlinear interaction term $[a^{\dagger 2}b^2 + {\rm h.c.}]$ is different from the Kerr nonlinear interaction $a^{\dagger 2}a^2$ and $a^\dagger a b^\dagger b$ [32, 33], as the latter is in the form of the square of the free Hamiltonian, and hence either $a^\dagger a$ or $b^\dagger b$ is a constant of motion.

We have performed extensive numerical simulations with the full Hamiltonian (1) and an initial state $|g\rangle \otimes |\psi\rangle$, i.e. the atom is initially in its ground state while the two cavity modes are in a specific state $|\psi\rangle$ (this will be specified later on). Ignoring the atomic spontaneous emission and the cavity decay, we find the above analytical insights to be completely accurate, i.e. we indeed get the maximal entangled state (see figure 2). In fact, we find that the approximated Hamiltonian (3) is quite a good approach to the full Hamiltonian (1).

The top panel in figure 2 shows selected results for the dependence of the population of state $|0, 2\rangle$ (dotted line) and $|2, 0\rangle$ (dashed line) on time, while the lower panel displays the Wootters concurrence which was used to measure the entanglement between the two modes in Fock states in this case. An initial state $|0, 2\rangle$ and system parameters $\Delta = 20g_c$, $\delta = 5g_c$ are chosen for this plot. The maximal entangled state may be obtained at time $t = (785/g_c)$ s with perfect fidelity (>99.9%).[3] The fidelity is defined as the overlap between the output state

[3] Even with currently available parameters of $g \sim (2\pi)100$ (MHz), the time to get a maximally entangled state is expected to 8×10^{-6} s. For an optical cavity with $Q \sim 10^9$, the photon lifetime is long enough for this protocol.

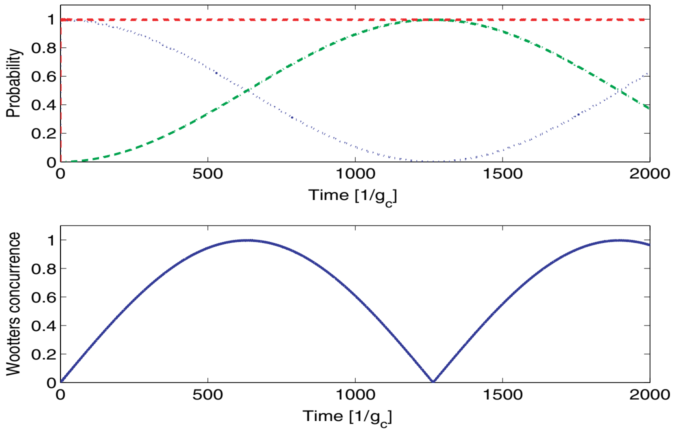

Figure 2. Populations of states $|0, 2\rangle$ (dotted line) and $|2, 0\rangle$ (dashed line), and the Wootters concurrence versus time; this figure is plotted for the case when the adiabatic limit is satisfied. The horizontal line in the top panel denotes the probability of the atom being in its ground state $|g\rangle$, regardless of where the photons are.

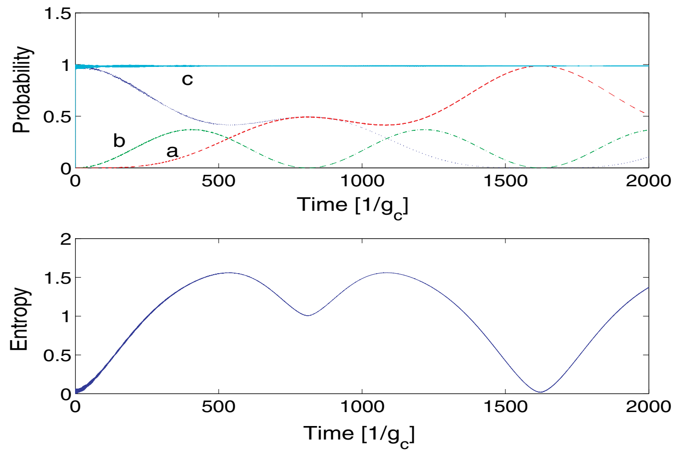

Figure 3. Top panel: the dependence of the population on states $|4, 0\rangle$ (line c), $|2, 2\rangle$ (line b) and $|0, 4\rangle$ (line a) on time with initial state $|4, 0\rangle$. Bottom panel: the von Neumann entropy versus time with the same initial state as in the top panel. The constant line in the top panel shows the population of the atom in the ground state $|g\rangle$, which indicates that the two cavity modes are near perfectly pure states, hence the quantum entropy is a good measure for the entanglement.

and the desired state. We would like to note that the two cavity modes are near perfectly pure states. This was shown by the horizontal line in the top panel of figure 2, which indicates that the population of atoms remains unchanged during the whole preparation process.

Similar results are found for the initial state $|4, 0\rangle$, illustrated in figure 3. The difference is that there are three components $|4, 0\rangle$, $|2, 2\rangle$ and $|0, 4\rangle$ in the output state; their populations are shown in the top panel in figure 3 by c, b and a, respectively. It is interesting to note that there is a time point(in figure 3, top panel) when line c and line a overlap, which corresponds to the system in the state $\frac{1}{\sqrt{2}}(|0, 4\rangle + |4, 0\rangle)$, and at this point the entropy as plotted in the lower panel is 1; it equals the entropy of a maximally entangled state for a two-qubit. We would like

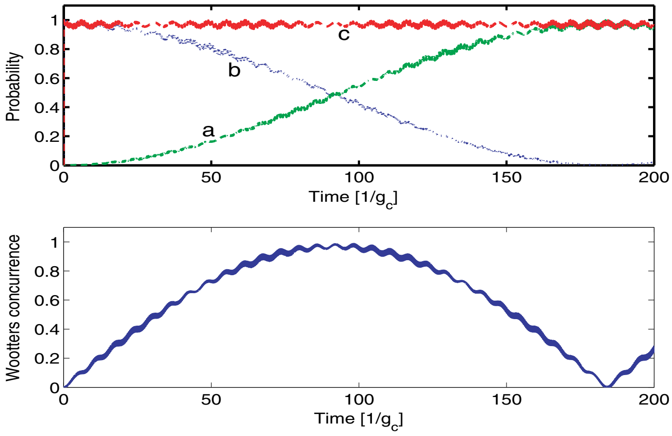

Figure 4. The same results as in figure 2, but for the case where adiabatic elimination of the atomic levels is not valid.

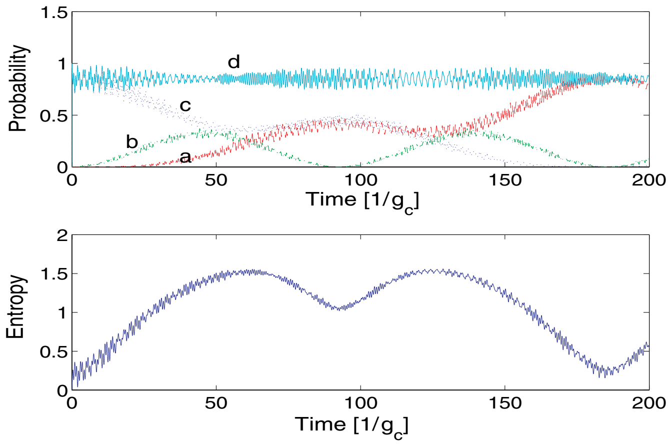

Figure 5. The same results as in figure 3, but for the case where adiabatic elimination of the atomic levels is not valid.

to point out that the Wootters concurrence is not a good measure for the entanglement in this situation, so we choose the von Neumann entropy instead to quantify the entanglement, which is the same for figure 7. A recent study on the two entangled modes show that the amount of entanglement present in a given state depends on how one defines one's systems [34]. This means we could redefine our two modes such that the state $\frac{1}{\sqrt{2}}(|0, 4\rangle + |4, 0\rangle)$ represents a maximally entangled state.

It is surprising to find that the same dynamics as in the adiabatic limit persists even when adiabatic elimination is not valid. As an example, in figures 4 and 5, we display results for $\Delta = 8g_c, \delta = 3g_c$. Apparently, the atom as the interaction agent for the two cavity modes is enough to establish an effective interaction between them. As figure 4 shows, the entanglement measured as the Wootters concurrence arrives at its maximum at the cross point of lines a and b (in the top panel) where the modes are maximally entangled. The same feature is found in figures 5, 6 and 7.

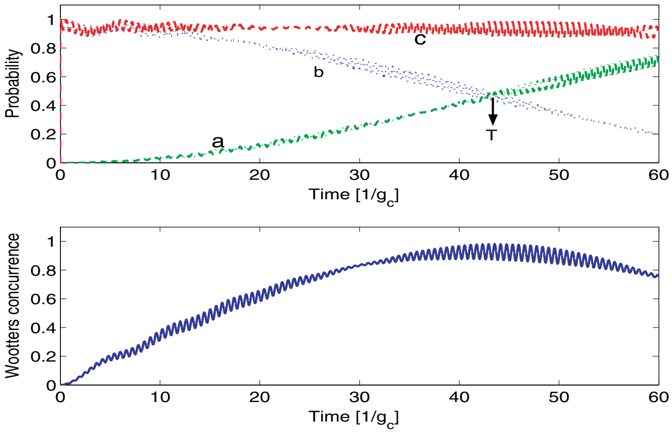

Figure 6. Populations for states $|2, 0\rangle$ (line a) and $|0, 2\rangle$ (line b) (top panel), as well as the Wootters concurrence, with the cavity decay rate $\kappa = 0.01g_c$ and $\Gamma = 0.3g_c$; the other parameters chosen are the same as in figure 4. The arrow in the figure indicates the instant when the modes got maximally entangled.

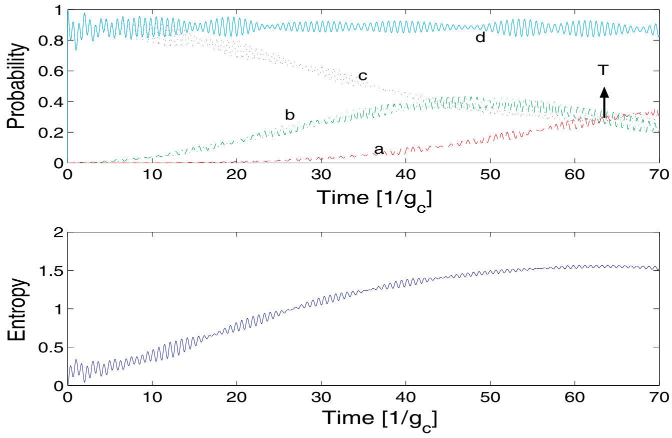

Figure 7. The same as in figure 5, but with the cavity decay rate $\kappa = 0.01g_c$ and $\Gamma = 0.3g_c$. The rapid oscillations appearing on these lines result from the population transfers among atomic states; these population transfers might be caused by the cavity decay and the diversion from the adiabatical elimination condition. At time T as indicated in the figures, the two modes would arrive at maximally entangled states.

Now, we discuss the effects of the dissipation or decoherence due to both the atomic decay and the cavity loss. As with any proposal for quantum information processing, ultimately its success depends on being able to complete many coherent dynamics during the decoherence time. In principle, as long as (a) $\frac{\lambda^2}{\delta} \gg \kappa$ and (b) $\frac{\lambda^2}{\delta} \gg \Gamma$, we could expect essentially the same results as illustrated in figures 2 and 3. As there are small virtual transitions of atomic states in our proposal, it makes this scheme immune to the atomic spontaneous emission or atomic decay, so the restriction (b) makes loss constriction in this scheme. Actually, we can see this point from the numerical calculation presented in figures 6 and 7. On the other hand, condition (a) is difficult to achieve because the two-photon process is relatively weak due to large off-resonant detunings for all its intermediate states. In figures 6 and 7, the effects of cavity decay as well as atomic decay on the dynamics of the proposed system is illustrated. As known, the decoherence time for a state $|m, n\rangle$ depends on the total number of photons, and as figures 6 and 7 show, relative good results are found when the cavity loss rate κ is small with relatively large atomic decay $\Gamma = 0.3g$.

Finally, we want to stress that the requirement for the intermediate states to be degenerate is not necessary. In fact, our proposal works in the same manner when the detunings Δ_i defined by $\Delta_i = \Omega - \omega_i$ have different sign, i.e. $\Delta_1 = -\Delta_2$. Although there are many cavity QED-based quantum computation protocols, the optical cavity with high-Q and an atom with small decay rate remain challenging because of the technological limit of the Fabry–Perot optical cavity [35–39]. The initial two-photon Fock state in a cavity mode can be prepared efficiently by a third-order Raman process [25]. With the same technology, Fock states with higher photon number can also be reached [25, 40, 41]. In contrast to the flying qubit entanglement, the two entangled cavity modes have relatively good location, and hence might be used to distribute entanglement among different nodes in a quantum network. Besides, the entangled modes could be coupled out of the cavity as an information carrier. This may allow us to extend quantum cryptography over long distances.

In conclusion, we have proposed a new protocol for preparing the maximally entangled state in two high-Q optical cavities. As the photons act as the information carrier, a cavity with very high-Q factor is highly desirable. This proposal works in the same way for one cavity with two modes, too. We have explained the scheme in terms of the virtual two-photon process induced nonlinear interaction. A similar idea can be found in [42] where large detunings from the sideband were used to diminish real populations of the system on the vibrational states, and hence the scheme is immune to decoherence due to the thermal noise. In addition, our protocol can also be explained in terms of entanglement distribution by separable states [43]: there a third particle that is never entangled with the other two was used to entangle the two particles. In our case the atom would act as the third particle as it was eliminated out of the system and to this extent it was never entangled with the two modes in the process, instead making the two modes entangled. This new protocol has the advantage that its ideal implementation involves no real transitions of atomic states; it makes this proposal immune to the atomic spontaneous emission or atomic decay. It is easy to map our model to an ion trap setup, where the trapped ion takes the role of the atom while their collective vibrational modes (say along the x- and y-axes) play the role of the two cavity modes. The long-lived vibrational quanta met the requirement of the protocol very well. In addition, this protocol provided a new kind of nonlinear interaction among the cavity modes, which might be useful for people interested in cavity-QED system.

Acknowledgments

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